Mathematical Analysis of the Effects of some Selected Factors on the Crude Divorce Rate by using ANN and ODE

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ABSTRACT

In this study, the factors affecting the Crude divorce rate in Turkey were determined as Total Fertility Rate (Number of Children), Crude Marriage Rate (Per Thousand(‰)), Population Growth Rate (Per Thousand(‰)) and General Happiness Level (%) (Unhappy or Very Unhappy). The effects of these factors were examined mathematically through both ANN and linear differential equations. For this purpose, the data obtained from TSI (Turkish Statistical Institute) for the years 2005-2023 was increased with the linear differential equation system for the same years and made suitable for ANN (Artificial Neural Network) analysis. Then, the performances of ANN activation function and differential equation system for crude divorce rate were compared. In this comparison, it is shown and predictions are made that linear ODE (Ordinary Differential Equation) system performs much better.

Keywords: ANN Analysis, Mathematical Modeling, Crude Divorce Rate, Ordinary Differential Equations.

1. INTRODUCTION

In today's world where divorce, which is an undesirable situation in society, is rapidly increasing, it is important to examine the factors affecting the reasons for divorce. According to the Turkish Statistical Institute, the number of marriages has decreased over the years, while the number of divorces has increased. Preventing divorces depends on a thorough investigation of the factors affecting divorces [1].

The main reasons for divorce can be listed as follows: family characteristics (socio-economic, separated parents), current characteristics (education, social class, housing ownership, economic activity, religion, sect), marital factors (age at marriage, premarital relationship status, previous relationship breakdown, child experience, time, age and number of children), interpersonal behavioral problems and divorce attitudes [2]. The crude divorce rate, which expresses the number of divorces per thousand population in Turkey, was 2.01 per thousand in 2023 [3]. In this study, the factors affecting the Crude divorce rate in Turkey were determined as Total Fertility Rate (Number of Children), Crude Marriage Rate (Per Thousand(‰)), Population Growth Rate (Per Thousand(‰)) and General Happiness Level (%) (Unhappy or Very Unhappy). The effects of these factors were examined mathematically through both ANN and linear differential equations.

Artificial intelligence means that certain behaviors specific to the human brain are imitated by machines. The use of artificial intelligence in many areas is becoming widespread very quickly. Artificial Neural Network (ANN) is a very simplified model of the neural network structure in human brain cells. The main basis of this structure is neurons. While a neuron can receive many inputs (dendrites), it can only give one output (axon). The output of one neuron can be the input of another neuron. Each neuron has different and variable activation thresholds. This complex network structure is usually formed by layers and different numbers of neurons in these layers. With the artificial intelligence network created according to each problem, complex non-linear problems that classical (logistic regression) statistical approaches are lacking can be solved [4]. In this study, ANN is one of the optimization algorithms whose performance is compared regarding the crude divorce rate.

Nowadays, scientists try to explain everything in mathematical terms in order to better understand the events around us and to produce solutions to technical problems related to them. Mathematical modeling can be defined as the process of describing a real-world problem in mathematical terms, usually in the form of equations, and then using those equations to both understand the original problem and discover new properties about the problem [5]. This process is done through differential equations, especially when the change in the magnitude of a variable with time is involved. Mathematical modeling through differential equations such as ordinary [6], fractional-order [7], impulsive [8], etc. has long been widely used in many areas of physics, chemistry, biology, medicine, economics and social sciences. In this study, another optimization technique whose performance is compared according to the crude divorce rate is the linear ordinary differential equation (ODE) system.

With the help of the data determined from TSI as total fertility rate (number of children), crude marriage rate (per thousand(‰)) population growth rate (per thousand(‰)) and general happiness level (%) (unhappy or very unhappy) between 2005-2023, crude divorce rate was determined with ANN and linear ODE optimization methods and a comparison was made. Since the data obtained from TSI was insufficient for ANN analysis, it was increased to 109 data between 2005 and 2023 with the help of linear ODE. Then, the divorce rate was calculated with ODE and ANN management and a comparison of these methods was made.

2. LITERATURE REVIEW

Some of the studies conducted using ANN and differential equations in the literature on divorce analysis can be summarized as follows.

In the study by Sadiq Fareed et al. [9], the authors introduced a new ensemble learning method that incorporates advanced machine learning techniques. They utilized algorithms such as Support Vector Machine (SVM), Passive Aggressive Classifier, and Neural Network (MLP) to predict divorce. A domain expert developed a question-based dataset, where the responses provided valuable insights into the likelihood of a marriage ending in divorce. They performed a 5-fold cross-validation to assess performance, yielding a 100% accuracy score. Additionally, metrics like the Receiver Operating Characteristic (ROC) curve accuracy, recall, precision, and F1 accuracy scores were all approximately 97%. Their research highlighted the crucial factors in predicting divorce and identifying key indicators.

Yöntem et al. [10] employed the Divorce Indicators Scale, derived from Gottman couple therapy, to predict divorce. Among the participants, 84 (49%) were divorced, while 86 (51%) were still married. The dataset for the study was created by administering this scale to the participants. The study evaluated the effectiveness of the scale using Artificial Neural Network, RBF Neural Network, and Decision Tree algorithms. It also aimed to identify the key items from the Divorce Indicators Scale. By applying a correlation-based feature selection method to the dataset, the six most influential features and their importance values were determined. The RBF neural network achieved the highest success rate of 98.23% when direct classification methods were applied to the dataset. However, when classification methods were applied to the six selected features, the Artificial Neural Network achieved the highest success rate of 98.82%.

Sharma et al. [11] conducted a study on predicting divorce cases using various machine learning algorithms. They implemented several classifiers, including Perceptron, Decision Tree, Random Forest, Naive Bayes, K-Nearest Neighbor, and Support Vector Machine, to predict divorce and compared their performances to identify the most accurate model. The study utilized the Gottman method as an approach. After training the algorithms, they were able to predict the likelihood of divorce, with the Perceptron model achieving the highest accuracy rate of 98.5%.

Afthanorhan et al. [12] explored the effectiveness of three machine learning techniques—Decision Tree, Logistic Regression, and Artificial Neural Network—in predicting divorce among Malaysian women. The study used secondary data from the Fifth Malaysian Population and Family Survey (MPFS-5), conducted by the National Population and Family Development Board (LPPKN), which included 7,644 married women aged 15 to 59. In Malaysia, the divorce rate increased by 12% in 2019 compared to the previous year. The researchers made comparisons with six predictive models: Decision Tree (C5.0 and CHAID), Logistic Regression (Forward and Backward Stepwise), and Artificial Neural Network (Multilayer Perceptron and Radial Basis Function). Of these, the Decision Tree (C5.0) performed the best, with an accuracy of 77.96%, followed by the Artificial Neural Network (Multilayer Perceptron) at 74.68% and Logistic Regression (Forward Stepwise) at 67.89%. The study also identified the key predictors of divorce among Malaysian women, ranking them as follows: spouses' employment status (0.1531), husbands' employment status (0.1396), marriage type (0.1327), race/ethnicity (0.1327), long-distance relationships (0.1212), spouses' educational level (0.1115), age group (0.1053), and religion (0.0998).

Gambrah and Adzadu [13] developed a compartmental mathematical model, using a system of nonlinear differential equations, to analyze the dynamics of the divorce epidemic. The authors examined the existence and stability of both divorce-free and endemic equilibrium states in relation to a threshold parameter. The global stability of these equilibria was assessed through the application of the Lassalle invariance principle of Lyapunov functions. Their findings indicated that reducing interactions between married and divorced individuals, increasing the stability of marriages, and providing education to those at risk of separation could help mitigate the divorce epidemic.

In [14], Tessema et al. introduced a deterministic model to explore marital divorce within a population, conducting a qualitative analysis based on the stability theory of differential equations. By employing the next-generation matrix method, they calculated the basic reproduction number in relation to the divorce-free equilibrium. The study identified conditions for both local and global asymptotic stability of divorce-free and endemic equilibria. The model displayed backward bifurcation, and they calculated the sensitivity indices of parameters influencing the spread or reduction of divorce.

Chang et al. [15] introduced a new method for analyzing divorce dynamics and strategies for mitigating it using nonlinear differential equations. Their model focused on parameter estimation, solution existence and uniqueness, positivity, boundedness, and invariant regions within the differential equation system. They analyzed the equilibria and stability conditions of the model, performing a sensitivity analysis, and provided recommendations for understanding and reducing divorce rates through targeted interventions.

In this study, the estimation of the magnitude of the crude divorce rate for Turkey is proposed through ANN and ODE. It is aimed to provide a different perspective to the literature both in terms of its application to the subject and in terms of comparing the performances of these two optimization methods.

3. METHODOLOGY

Within the scope of this study, the data obtained from the Turkish Statistical Institute regarding the crude divorce rate between 2005-2023 were analyzed and the features affecting these problems were examined mathematically with both ANN and ODE. 4 features were given as input to the model and an artificial neural network model was used to estimate the crude divorce rate.

The first stage of the model is to organize and complete the missing data obtained from TSI (19 rows). When the data is analyzed, it is seen that each input parameter contains data for the years 2005-2023. Since the data for the ANN analysis is insufficient, the variables were modeled with the general linear ordinary differential equation system and these data were increased (109 rows). Using this system, the missing data for each input was completed and data was estimated for the same time period but with intermediate values added. Thus, a data set of 109x5 size covering the years 2005-2023 was obtained.

In the second stage of the designed system, it was aimed to estimate the crude divorce rate using the data set obtained as the output of the ODE system. However, in this data set, the only data that is not true is the data obtained with ODE. Because in the performance comparison, ANN and ODE results were compared. Therefore, an original Artificial Neural Network model was designed. A total of 109 data were divided into three as training, testing and validation and used in the training and testing processes of the ANN model. Thus, the process of revealing the model that estimates the crude divorce rate with the lowest error rate was carried out with comparative analysis.

3.1. Dataset

In this study, the effects of some selected factors on the Crude Divorce Rate for Turkey were mathematically examined with the help of data obtained from TSI. For this purpose, the state variables used to represent the time parameter t are shown in Table 1.

Table 1. State Variables

Variable	Definition
$x(t)$	Total Fertility Rate (Number of Children)
$y(t)$	Crude Marriage Rate (Per Thousand (‰))
$z(t)$	Population Growth Rate (Per Thousand (‰))
$u(t)$	General Happiness Level (%) (Unhappy or Very Unhappy)
$v(t)$	Crude Divorce Rate (Per Thousand (‰))

The time-dependent values of the variables used for the years 2005-2023 taken from TSI are presented in Table 2 [3].

Table 2. Dataset

t	$x(t)$	$y(t)$	$z(t)$	$u(t)$	$v(t)$
2005	2,12000	9,37002	5,72782	12,86000	1,40
2006	2,12000	9,17986	5,69520	11,89000	1,35
2007	2,16000	9,09818	5,66295	11,06000	1,34
2008	2,15000	9,03530	13,18733	13,90000	1,40
2009	2,09990	8,21417	14,60087	14,60000	1,58

t	$x(t)$	$y(t)$	$z(t)$	$u(t)$	$v(t)$
2010	2,08018	7,96688	16,00958	10,75000	1,62
2011	2,04719	7,98634	13,58167	9,88000	1,62
2012	2,10861	8,03119	12,08597	10,16000	1,64
2013	2,10681	7,88124	13,75798	10,75000	1,65
2014	2,18618	7,77001	13,40901	11,73000	1,70
2015	2,15709	7,70895	13,45179	11,39000	1,69
2016	2,11484	7,49884	13,63733	10,41000	1,59
2017	2,08180	7,09052	12,47454	11,05000	1,60
2018	2,00473	6,81007	14,76735	12,11000	1,76
2019	1,88783	6,56718	14,03732	13,06000	1,90
2020	1,76974	5,85641	5,52420	14,50000	1,64
2021	1,70964	6,69231	12,74794	16,62000	2,09
2022	1,63106	6,77679	7,07697	15,86000	2,15
2023	1,50935	6,62676	1,08847	13,70000	2,01

3.2. Data Augmentation with ODE Model

Since the data set for ANN analysis was insufficient, this data set was modeled as a linear differential equation system and increased by finding intermediate values for the years 2005-2023. However, for this, min-max normalization was first applied to the data. One of the most commonly used methods for normalising data is min-max normalisationFor each feature, the lowest value is set to 0 and the highest value is set to 1. All other values are then adjusted to fall between 0 and 1 as decimals. Thus, the data set for a variable x is created with the formula

$$x_{norm} = \frac{x - x_{min}}{x_{max} - x_{min}}. \quad (1)$$

Some results obtained from the data set in Table 2 for the normalization process are given in Table 3.

Table 3. Some results for normalizing the dataset

	t	$x(t)$	$y(t)$	$z(t)$	$u(t)$	$v(t)$
min	2005	1,50935	5,85641	1,08847	9,88	1,34
max	2023	2,18618	9,37002	16,00958	16,62	2,15
max-min	18	0,67683	3,51361	14,92111	6,74	0,81

The data set in Table 2, normalized with min-max normalization, is presented in the Table.

Table 4. Normalized Dataset

t_{norm}	$x_{norm}(t)$	$y_{norm}(t)$	$z_{norm}(t)$	$u_{norm}(t)$	$v_{norm}(t)$
0,00000	0,90222	1,00000	0,31093	0,44214	0,07407
0,05556	0,90222	0,94588	0,30874	0,29822	0,01235
0,11111	0,96132	0,92263	0,30658	0,17507	0,00000
0,16667	0,94654	0,90474	0,81085	0,59644	0,07407
0,22222	0,87252	0,67104	0,90559	0,70030	0,29630
0,27778	0,84339	0,60066	1,00000	0,12908	0,34568
0,33333	0,79465	0,60619	0,83728	0,00000	0,34568
0,38889	0,88539	0,61896	0,73704	0,04154	0,37037
0,44444	0,88273	0,57628	0,84910	0,12908	0,38272
0,50000	1,00000	0,54463	0,82571	0,27448	0,44444
0,55556	0,95702	0,52725	0,82858	0,22404	0,43210
0,61111	0,89459	0,46745	0,84101	0,07864	0,30864
0,66667	0,84578	0,35124	0,76308	0,17359	0,32099
0,72222	0,73191	0,27142	0,91675	0,33086	0,51852
0,77778	0,55919	0,20229	0,86782	0,47181	0,69136
0,83333	0,38472	0,00000	0,29728	0,68546	0,37037
0,88889	0,29592	0,23790	0,78141	1,00000	0,92593
0,94444	0,17982	0,26195	0,40134	0,88724	1,00000
1,00000	0,00000	0,21925	0,00000	0,56677	0,82716

The model in the form of a linear differential equation system used for data augmentation is

$$\begin{aligned}
\frac{d\bar{x}}{d\bar{t}} &= \alpha_1 + \alpha_2\bar{x} + \alpha_3\bar{y} + \alpha_4\bar{z} + \alpha_5\bar{u} + \alpha_6\bar{v} \\
\frac{d\bar{y}}{d\bar{t}} &= \alpha_7 + \alpha_8\bar{x} + \alpha_9\bar{y} + \alpha_{10}\bar{z} + \alpha_{11}\bar{u} + \alpha_{12}\bar{v} \\
\frac{d\bar{z}}{d\bar{t}} &= \alpha_{13} + \alpha_{14}\bar{x} + \alpha_{15}\bar{y} + \alpha_{16}\bar{z} + \alpha_{17}\bar{u} + \alpha_{18}\bar{v} \\
\frac{du}{d\bar{t}} &= \alpha_{19} + \alpha_{20}\bar{x} + \alpha_{21}\bar{y} + \alpha_{22}\bar{z} + \alpha_{23}\bar{u} + \alpha_{24}\bar{v} \\
\frac{dv}{d\bar{t}} &= \alpha_{25} + \alpha_{26}\bar{x} + \alpha_{27}\bar{y} + \alpha_{28}\bar{z} + \alpha_{29}\bar{u} + \alpha_{30}\bar{v} \\
\bar{x}(\bar{t}_0) &= \bar{x}_0, y(\bar{t}_0) = \bar{y}_0, \bar{z}(\bar{t}_0) = \bar{z}_0, \bar{u}(\bar{t}_0) = \bar{u}_0, \bar{v}(\bar{t}_0) = \bar{v}_0.
\end{aligned} \tag{2}$$

where $t_{norm} = \bar{t}$, $x_{norm}(\bar{t}) = \bar{x}$, $y_{norm}(\bar{t}) = \bar{y}$, $z_{norm}(\bar{t}) = \bar{z}$, $u_{norm}(\bar{t}) = \bar{u}$ and $v_{norm}(\bar{t}) = \bar{v}$ in Table 4. Model in (2) was solved with Matlab R2023a program rungekutta45 and then the parameter values closest to the values in Table 4 (giving the minimum error) were found using the lsqcurvefit function (Mathworks, 2024) (Işık & Daşbaşı, 2023). In this context, the graphical results of the approach used are in Figures 1-5. In this context, the parameters a_i for $i = 1,2,\dots,30$ obtained for are given in Table 5.

Table 5. Rate Constants

$\alpha_1 = -7.31434$	$\alpha_2 = -0.14969$	$\alpha_3 = 9.26094$	$\alpha_4 = 0.06112$	$\alpha_5 = -2.38716$
$\alpha_6 = 6.81211$	$\alpha_7 = 21.56334$	$\alpha_8 = -19.54863$	$\alpha_9 = -0.28976$	$\alpha_{10} = 0.31362$
$\alpha_{11} = -12.06159$	$\alpha_{12} = -8.40744$	$\alpha_{13} = 15.52661$	$\alpha_{14} = -3.27666$	$\alpha_{15} = -9.13576$
$\alpha_{16} = -0.52615$	$\alpha_{17} = -4.15354$	$\alpha_{18} = -17.18764$	$\alpha_{19} = 26.74022$	$\alpha_{20} = 8.09197$
$\alpha_{21} = -34.99948$	$\alpha_{22} = -5.70690$	$\alpha_{23} = -0.44531$	$\alpha_{24} = -26.88373$	$\alpha_{25} = -4.08232$
$\alpha_{26} = -10.95642$	$\alpha_{27} = 7.90501$	$\alpha_{28} = 11.83036$	$\alpha_{29} = 2.69993$	$\alpha_{30} = -0.67234$

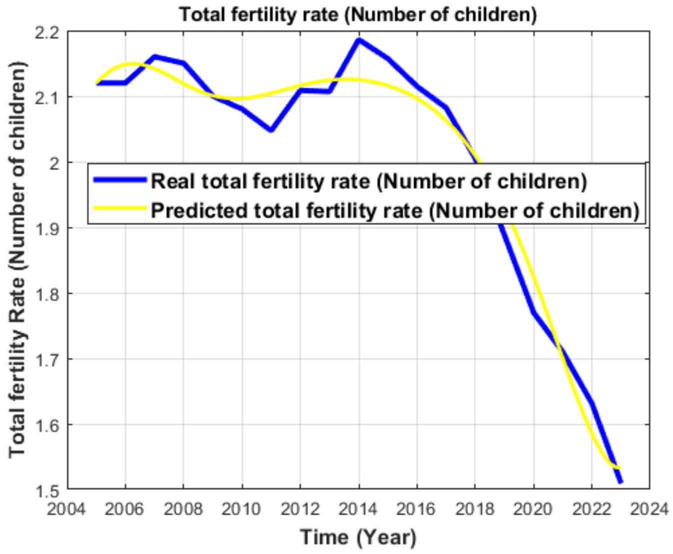

Figure 1. The fitted ODE curve compared to the true value of $x(t)$

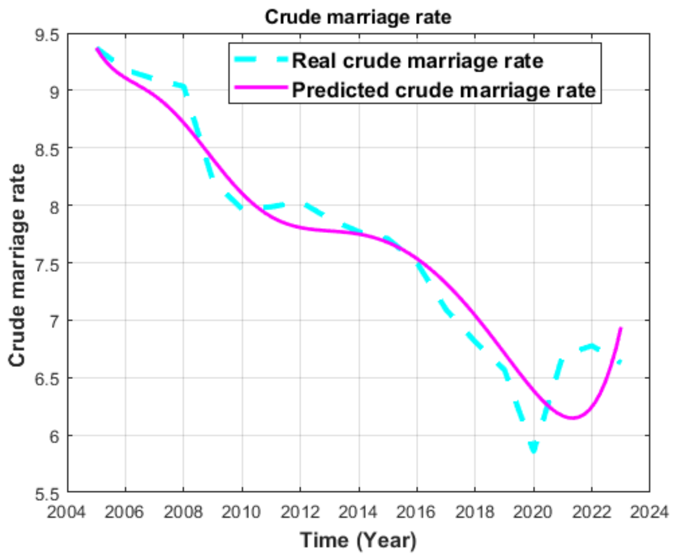

Figure 2. The fitted ODE curve compared to the true value of $y(t)$

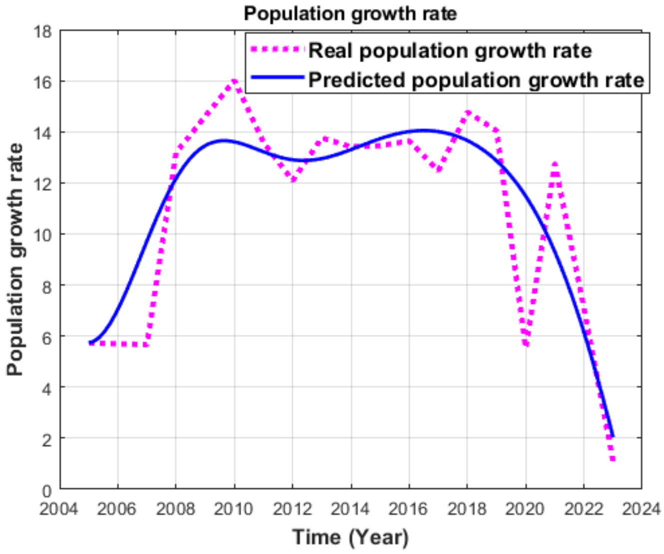

Figure 3. The fitted ODE curve compared to the true value of $z(t)$

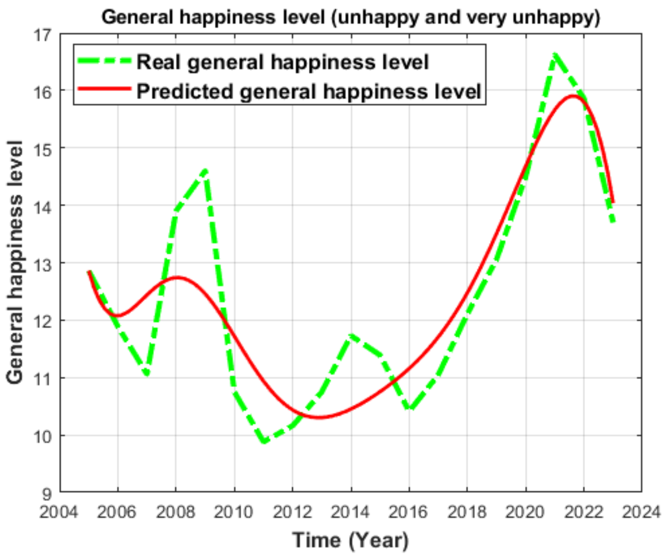

Figure 4. The fitted ODE curve compared to the true value of $u(t)$

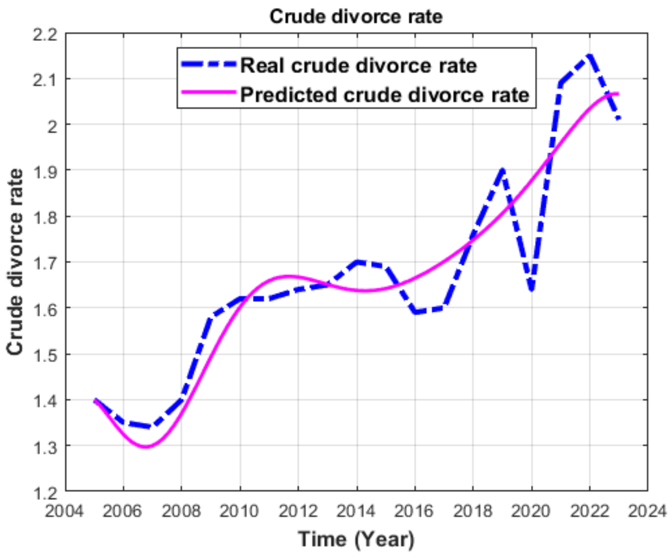

Figure 5. The fitted ODE curve compared to the true value of $v(t)$

Data were estimated with the ODE system and the augmented data set from which the estimated results were obtained is shown in Table 6.

Table 6. Augmented Data

t	$x(t)$	$y(t)$	$z(t)$	$u(t)$	$v(t)$	t	$x(t)$	$y(t)$	$z(t)$	$u(t)$	$v(t)$
2005	2,12	9,370022	5,727823	12,86	1,4	2014	2,186179	7,77001	13,40901	11,73	1,7
2005,2	2,1281	9,2988	5,7957	12,566	1,3883	2014,1	2,1244	7,745	13,3393	10,4735	1,6375
2005,4	2,135	9,2412	5,9475	12,3491	1,3744	2014,3	2,1233	7,7353	13,4193	10,522	1,6371
2005,5	2,1405	9,1944	6,1773	12,2009	1,3595	2014,5	2,122	7,7237	13,4994	10,5746	1,6374
2005,7	2,1446	9,156	6,4775	12,1129	1,3446	2014,6	2,1203	7,7101	13,5781	10,6308	1,6384
2005,9	2,1475	9,1235	6,8393	12,076	1,3307	2014,8	2,1183	7,6943	13,6543	10,6904	1,6399
2006	2,12	9,179864	5,695202	11,89	1,35	2015	2,15709	7,708946	13,45179	11,39	1,69
2006,1	2,1491	9,095	7,253	12,0813	1,3185	2015,2	2,1133	7,6559	13,7935	10,8191	1,645
2006,3	2,1495	9,0685	7,7077	12,1196	1,3085	2015,4	2,1102	7,6332	13,8543	10,8881	1,6485
2006,5	2,1488	9,0424	8,1927	12,1822	1,3015	2015,5	2,1068	7,608	13,9078	10,9604	1,6525
2006,6	2,1472	9,0155	8,6973	12,2611	1,2975	2015,7	2,1029	7,5804	13,9536	11,0363	1,6571
2006,8	2,1448	8,9866	9,2114	12,3489	1,2969	2015,9	2,0987	7,5503	13,9909	11,1163	1,6621
2007	2,16	9,098178	5,66295	11,06	1,34	2016	2,114836	7,498843	13,63733	10,41	1,59
2007,2	2,1381	8,9201	10,2275	12,5237	1,306	2016,1	2,0939	7,5177	14,0191	11,2006	1,6677
2007,4	2,1341	8,8815	10,7122	12,5996	1,3155	2016,3	2,0887	7,4827	14,0376	11,2899	1,6736
2007,5	2,1298	8,8391	11,1718	12,6622	1,3281	2016,5	2,083	7,4454	14,046	11,3845	1,68
2007,7	2,1254	8,793	11,6002	12,7081	1,3433	2016,6	2,0768	7,4056	14,0436	11,4852	1,6867
2007,9	2,1211	8,7436	11,9921	12,7348	1,3609	2016,8	2,0699	7,3636	14,0302	11,5922	1,6938
2008	2,15	9,035297	13,18733	13,9	1,4	2017	2,081796	7,090523	12,47454	11,05	1,6
2008,1	2,1168	8,691	12,3437	12,7405	1,3805	2017,2	2,0544	7,2728	13,9692	11,828	1,7089
2008,3	2,1129	8,6359	12,6529	12,7247	1,4016	2017,4	2,0456	7,2242	13,9211	11,9578	1,717
2008,5	2,1092	8,5788	12,9186	12,6876	1,4239	2017,5	2,0361	7,1735	13,8606	12,0959	1,7254

t	$x(t)$	$y(t)$	$z(t)$	$u(t)$	$v(t)$	t	$x(t)$	$y(t)$	$z(t)$	$u(t)$	$v(t)$
2008,6	2,1059	8,5202	13,1405	12,6297	1,4468	2017,7	2,0258	7,1209	13,787	12,2425	1,7342
2008,8	2,103	8,461	13,3191	12,5522	1,4699	2017,9	2,0147	7,0664	13,7002	12,398	1,7433
2009	2,099899	8,214166	14,60087	14,6	1,58	2018	2,00473	6,810073	14,76735	12,11	1,76
2009,2	2,0988	8,3432	13,5541	12,3458	1,5154	2018,1	2,0027	7,0102	13,5994	12,5625	1,7528
2009,4	2,0974	8,2859	13,616	12,2212	1,537	2018,3	1,9899	6,9525	13,4841	12,7359	1,7627
2009,5	2,0965	8,2304	13,6453	12,086	1,5574	2018,5	1,9762	6,8936	13,3536	12,918	1,7729
2009,7	2,0962	8,1773	13,6463	11,9429	1,5764	2018,6	1,9616	6,8336	13,2069	13,1081	1,7835
2009,9	2,0963	8,127	13,6232	11,7947	1,5938	2018,8	1,9461	6,7728	13,043	13,3056	1,7946
2010	2,08018	7,966884	16,00958	10,75	1,62	2019	1,887829	6,567179	14,03732	13,06	1,9
2010,1	2,0968	8,08	13,5802	11,6439	1,6094	2019,2	1,9123	6,6501	12,659	13,7186	1,8181
2010,3	2,0977	8,0365	13,5215	11,4932	1,6231	2019,4	1,8941	6,589	12,4365	13,9316	1,8305
2010,5	2,099	7,9966	13,4512	11,3448	1,635	2019,5	1,875	6,5288	12,192	14,147	1,8434
2010,6	2,1006	7,9606	13,374	11,2013	1,6449	2019,7	1,8551	6,4699	11,9245	14,3631	1,8568
2010,8	2,1024	7,9286	13,2938	11,0648	1,653	2019,9	1,8344	6,4132	11,6326	14,5777	1,8706
2011	2,047192	7,986338	13,58167	9,88	1,62	2020	1,769739	5,856412	5,524202	14,5	1,64
2011,2	2,1065	7,876	13,1383	10,8194	1,6638	2020,1	1,813	6,3592	11,315	14,7882	1,8848
2011,4	2,1087	7,8551	13,0682	10,7127	1,6667	2020,3	1,791	6,3089	10,9704	14,9917	1,8993
2011,5	2,1109	7,8375	13,0063	10,6176	1,6683	2020,5	1,7686	6,2633	10,5975	15,185	1,9141
2011,7	2,1131	7,823	12,9547	10,5351	1,6686	2020,6	1,7458	6,2233	10,1949	15,3645	1,9291
2011,9	2,1152	7,8112	12,9149	10,4654	1,6678	2020,8	1,7228	6,1902	9,7613	15,526	1,9443
2012	2,10861	8,031185	12,08597	10,16	1,64	2021	1,709639	6,692311	12,74794	16,62	2,09
2012,1	2,1172	7,8017	12,888	10,4085	1,6661	2021,2	1,6772	6,1501	8,7981	15,7778	1,9743
2012,3	2,119	7,7943	12,8744	10,3644	1,6638	2021,4	1,6549	6,1459	8,2668	15,8582	1,9889
2012,5	2,1207	7,7884	12,8739	10,3324	1,6609	2021,5	1,6333	6,1546	7,7017	15,9009	2,003
2012,6	2,1222	7,7838	12,8863	10,3119	1,6577	2021,7	1,6128	6,1779	7,1027	15,8998	2,0164
2012,8	2,1234	7,7799	12,9112	10,3025	1,6544	2021,9	1,5936	6,2178	6,4703	15,8481	2,0289
2013	2,106811	7,881244	13,75798	10,75	1,65	2022	1,63106	6,77679	7,076973	15,86	2,15
2013,2	2,1251	7,7731	12,9946	10,3132	1,6478	2022,1	1,5763	6,2765	5,8049	15,7386	2,0401
2013,4	2,1255	7,7693	13,0509	10,3317	1,6448	2022,3	1,5611	6,3564	5,1076	15,5635	2,0498
2013,5	2,1257	7,7649	13,115	10,3578	1,6423	2022,5	1,5486	6,46	4,3798	15,3147	2,0577
2013,7	2,1256	7,7596	13,1855	10,3907	1,6401	2022,6	1,5393	6,5897	3,6235	14,9833	2,0633
2013,9	2,1251	7,7531	13,2609	10,4295	1,6385	2022,8	1,5337	6,7485	2,8413	14,56	2,0663
						2023	1,509347	6,626764	1,088467	13,7	2,01

In Table 6, the green colored data represent the data augmented by ODE, while the colorless data corresponding to integer years are the real data obtained from TSI.

3.3. Design of ANN Prediction Model and Results

The ANN model proposed in this study, whose schema is given in Figure 6, was used to estimate the crude divorce rate in Turkey using four different input parameters. The selection of activation functions is of critical importance in the design of the network and facilitates the discovery and learning of complex patterns and relationships found in the data by applying them to the output of each neuron. It also ensures that the network maintains its nonlinear structure. The layers of the network, the number of nodes in the layers, and the activation function selected in each layer directly affect the overall performance of the model. The structure proposed in this study is the network with the highest performance as a result of the tests.

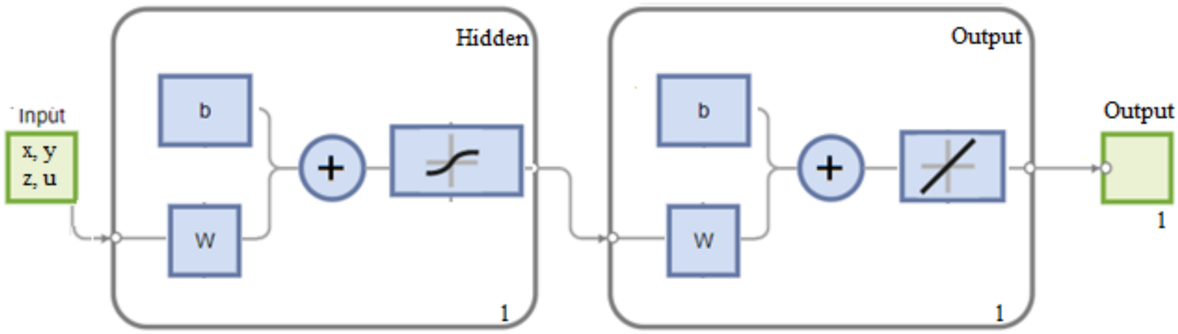

Figure 6. ANN architecture

In this structure, the variable v(t) was estimated using the other four inputs. For estimation, the training and testing processes were performed using the data in Table 6. No normalization was made to the data and the entire process was performed with the original data. Hyperbolic tangent transfer function was used as the activation function. In addition, 70% of the data was allocated for training, while 15% was set aside for validation and the remaining 15% for testing.

3.1.1. Parameter selection of ANN

Some parameters for ANN analysis are explained in Table 7. The model was determined by considering the parameter selections and the situations where good results were obtained.

Table 7. Training progress

Unit	Initial Value	Stopped Value	Target Value
Epoch	0	384	10000
Elapsed Time	-	00:00:02	-
Performance	2.24	0.00342	1e-16
Gradient	26.1	1.56e-09	0
Mu	0.001	1e+10	1e+10
Validation Checks	0	380	1.11e+06

We also have here:

-Best Validation Performance is 0.0020629 at epoch 4.

-Training: R=0.96139, Validation: R=0.96142, Test: R=0.97623 ve All: R=0.96218.

3.1.2. Activation function of ANN

The outcoming model by activation functions of ANN is written as in Equation (3). In this respect, it is

$$v(t) = v = b_2 + LW \tanh\left(b_1 + IW\begin{pmatrix} x \\ y \\ z \\ u \end{pmatrix}\right) \quad (3)$$

where

$$\begin{matrix} b_1 = (7.4216), b_2 = (1.8368), LW = (0.5036), \\ IW = (-3.4983 \quad -0.0566 \quad 0.0892 \quad -0.1124). \end{matrix} \quad (4)$$

4. Comparison of ANN and ODE Models and Discussion

In this section, the ANN and ODE model estimation results for the original values between 2005-2023 are shown in Table 8.

Table 8. Estimation performances of the proposed models according to the values in Table 2

t	Real v	The ANN prediction of v	The ODE prediction of v
2005	1,4	1,3847	1,4
2006	1,35	1,3975	1,3246
2007	1,34	1,3923	1,2998
2008	1,4	1,4537	1,3707
2009	1,58	1,5235	1,4929
2010	1,62	1,793	1,6016
2011	1,62	1,7908	1,6592
2012	1,64	1,6146	1,66695
2013	1,65	1,6558	1,651
2014	1,7	1,5055	1,638
2015	1,69	1,5517	1,6422
2016	1,59	1,6651	1,6649
2017	1,6	1,6488	1,7012
2018	1,76	1,826	1,74805
2019	1,9	1,9503	1,8061
2020	1,64	1,719	1,8777
2021	2,09	1,9974	1,9593
2022	2,15	1,9267	2,0345
2023	2,01	1,9943	2,0663

The performance indicators for both models compared to the actual values are presented in Table 9. According to these results, it is seen that both the ODE model and the ANN model are successful in estimating the crude divorce rate, but the ODE model is more successful.

Table 9. Performances of ODE and ANN estimation results

	The ANN prediction of v	The ODE prediction of v
Total Absolute Error	1,5839	1,1995
MAD	0,08336316	0,063132
MSE	0,01094062	0,007071
RMSE	0,10459743	0,084087
MAPE	4,90834872	3,673646
R-Squared	0,78554644	0,863293

5. CONCLUSION

Total absolute error is defined as the absolute value of the difference between the measured value and the true value of the measurement and is usually given as the highest possible error considering the degree of accuracy of a measurement instrument. The units of absolute error are the same as the measurement. Accordingly, the absolute error of the ODE estimate is 1.1995 measured and has a lower total error than ANN.

The mean absolute deviation (MAD) of a data set is the average distance between each data point and the mean. It gives us an idea of the variability in a data set. This value is obtained by dividing the total absolute error by the number of data. This value is 0.08336316 for ANN, while it is 0.063132 for ODE.

The mean square error (MSE) is the mean square difference between the observed value in a statistical study and the values predicted from a model. The RMSE value is found from the square root of this value. As the distance between the data points and the associated values from the model increases, the mean square error increases. Therefore, a model with a lower mean square error predicts the dependent values more accurately for the independent variable values. Although they are close to each other, ODE's performance is slightly better.

Mean absolute percentage error (MAPE), one of the most common metrics of model prediction accuracy, is the percentage equivalent of mean absolute error (MAE). Mean absolute percentage error measures the average magnitude of error produced by a model, or how far off the estimates are from the mean. ODE estimation is also more successful for this measurement tool.

In statistics, the coefficient of determination, pronounced as R-Square, is the proportion of the variation in the dependent variable that can be predicted from the independent variable(s). According to this value, ODE estimation is more successful.

Table 10. Crude divorce rate ODE estimate for 2024-2030

Year	ODE Estimate	Year	ODE Estimate	Year	ODE Estimate	Year	ODE Estimate	Year	ODE Estimate	Year	ODE Estimate	Year	ODE Estimate
2024	2,08696	2025	2,18358	2026	2,37867	2027	2,60031	2028	3,00485	2029	3,52174	2030	4,2504

As can be seen in Table 10, the crude divorce rate for Turkey is expected to be 4.2504 per thousand in 2024. Of course, this situation is directly related to the effect of the other 4 variables, especially the crude marriage rate. It is observed that the crude marriage rate is gradually decreasing. In addition, while the population growth rate was 5.72782 in 2005, it was measured as 1.08847 in 2023. These negative situations affect the crude divorce rate, which is already on the rise, even more negatively.

The results obtained in this study suggest that the current family structure in Turkey should be preserved and young people should be encouraged to marry and have children. It emphasizes that authorities should take precautions in these matters.

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