Solving the Cauchy and Boundary Value Problem for a Continuous Multiplicative Differential Equation of the Second Order

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ABSTRACT

In the presented work, the Cauchy and boundary value problem for the inhomogeneous equation with the second order multiplicative derivative in the case of discontinuity is considered. The general solution of the considered equation, which depends on two arbitrary constants, has been obtained. Then, when looking at the Cauchy problem, these arbitrary constants are determined from the given initial conditions, and when looking at the boundary problem, these arbitrary constants are determined from the given boundary conditions. Thus, analytical expressions have been obtained for the solution of the problems considered both in the Cauchy problem and in the boundary problem. Moreover, in the continuous case, an analytical expression has been obtained for the solution of the Cauchy and boundary problem considered for the second formulation multiplicative derivative inhomogeneous differential equation. Thus, all seven algebraic operations and integration operations are involved in this analytical expression.

Keywords: Uninterruptible multiplicative differential equation, multiplicative derivative, multiplicative integral, uninterruptible analysis

1. INTRODUCTION

It is known that the multiplicative integral without interruption was given by Voltaire in 1887 (Aliyev, 2014: 36, 45-49). later designated in more general terms (for large areas) by Birkhoff in 1937 (Birkhoff, 1937: 104-132).

In Gantmacher's book Theory of Matrices, this integral is given in the following form. (p. 434, formula (50)): $\int_{t_0}^{\hat{t}}[E+P(t)dt] = \lim_{\Delta t_k \to 0}[E+P(E_n)\Delta t_n]\dots[E+P(E_1)\Delta t_1]$.

and the multiplicative derivative on the same page

$$D_t X = \frac{dX}{dt}X^{-1},$$

given in the figure. We note that

$$D_t \text{ və } \int_{t_0}^{\hat{t}} ⬚$$

are mutually inverse actions. Note that the multiplicity property is not used for the definitions given by V. Volterra.Burada multiplikativ törəmə və inteqral üçün N.Ə. Əliyevin verdiyi təriflərdən istifadə ediləcəkdi [Aliyev, 2014:. 3, 45-49). – (Birkhoff, 1937 104-132).

MAİN RESULTS

Let's look at the issue as follows:

$$y^{[II]}(x) = f(x), \quad x > 0 \tag{1}$$

$$y(0) = \alpha, \qquad y^{[I]}(0) = \beta, \tag{2}$$

where f(x) is a real-valued continuous function and α and β are real constants. which is the multiplicative integral on both sides of equation (1), so that the integration is carried out over the piece [0, x]:

$$\int_0^x y^{[II]}(t)^{dt} = \int_0^x f(t)^{dt},$$

$$\frac{y^{[I]}(x)}{y^{[I]}(0)} = \int_0^x f(t)^{dt},$$

$$y^{[I]}(x) = y^{[I]}(0)\int_0^x f(t)^{dt}$$

Let's multiplicatively integrate the obtained expression again on [0, x]:

$$\int_0^x y^{[I]}(\tau)^{d\tau} = \int_0^x \left(y^{[I]}(0)\int_0^\tau f(t)^{dt}\right)^{d\tau},$$

since the multiplicative integral of the product is equal to the product of the integrals (this property does not exist for the definition of V. Volterra).

$$\frac{y(x)}{y(0)} = \int_0^x y^{[I]}(0)^{d\tau}\int_0^x\left(\int_0^{\tau\tau} f(t)^{dt}\right)^{d\tau} = \left(y^{[I]}(0)\right)^x\int_0^x\left(\int_0^\tau f(t)^{dt}\right)^{d\tau}$$

It is known that:

$$\int_0^\tau f(t)^{dt} = e^{\int_0^\tau \ln f(\tau)^{d\tau}}$$

then

$$\int_0^x\left(\int_0^\tau f(t)^{dt}\right)^{d\tau} = \int_0^x\left(e^{\int_0^\tau \ln f(t)^{dt}}\right)^{d\tau} = e^{\int_0^x d\tau \int_0^\tau \ln f(t)dt} =$$

$$= e^{\int_0^x \ln f(t)dt\cdot(x-t)} = e^{\int_0^x \ln f(t)^{x-t}dt}$$

Indeed:

$$\left(e^{\int_0^x \ln f(t)^{x-t}dt}\right)^{[I]} = \lim_{h\to 0}\sqrt[h]{\frac{e^{\int_0^{x+h}\ln f(t)^{x+h-t}dt}}{e^{\int_0^x \ln f(t)^{x-t}dt}}} =$$

$$= \lim_{h\to 0} \sqrt[h]{e^{\int_0^{x+h} \ln f(t)^{x+h-t}dt - \int_0^{x} \ln f(t)^{x-t}dt}} =$$

$$= \lim_{h\to h_0} \sqrt[h]{e^{\int_0^{x} \ln f(t)^{h}dt + \int_x^{x+h} \ln f(t)^{x+h-t}dt}} = e^{\int_0^{x} \ln f(t)^{d\tau}},$$

$$\left(e^{\int_0^x \ln f(t)dt}\right)^{[I]} = \lim_{h\to 0} \sqrt[n]{\frac{e^{\int_0^{x+h} \ln f(t)dt}}{e^{\int_0^{x} \ln f(t)dt}}} = \lim_{h\to 0} e^{\frac{l^{\int_0^{x+h} \ln f(t)dt - \int_0^{x} \ln f(t)dt}}{h}} =$$

$$= e^{\left(\int_0^x \ln f(t)dt\right)^{(I)}} = e^{\ln f(x)} = f(x)$$

Deməli:

$$y(x) = y(0) \cdot \left(y^{[I]}(0)\right)^{x} e^{\int_0^x \ln f(t)^{x-t}dt} \tag{3}$$

expression is a general solution of equation (1), y(0) and $y^{[I]}(0)$ are arbitrary constants.

Then (1), (2) solution of the Cauchy problem:

$$y(x) = \alpha\beta^{x} e^{\int_0^x \ln f(t)^{x-t}dt} \tag{4}$$

in the form of If considering equation (1) also t∈(0,l).

$$y(0) = \alpha, \quad y(l) = \beta \tag{5}$$

if we accept the boundary conditions, then from the general solution of (3) and the second boundary condition, we get:

$$\beta = \alpha \left(y^{[I]}(0)\right)^{e} e^{\int_0^l \ln f(t)^{l-t}dt} \tag{6}$$

From here too

$$y^{[I]}(0) = \sqrt[l]{\frac{\beta}{\alpha\, e^{\int_0^l \ln f(t)^{l-t}dt}}} = \sqrt[l]{\frac{\beta}{\alpha} e^{-\int_0^l \ln f(t)^{l-t}dt}} \tag{7}$$

it is taken as is. At this time (1), (5) solving the border issue: (Mamiyeva, 2019: 58-64)– (Mamiyeva, 2016: 121-124)

$$y(x) = \alpha \left(\frac{\beta}{\alpha} e^{-\int_0^l \ln f(t)^{l-t}dt}\right)^{\frac{x}{l}} \cdot l^{\int_0^x \ln f(t)^{x-t}dt} \tag{8}$$

is taken in the form of.

Note 1: Since the solution of equation (6) with respect to is not the only one, the solution of the considered boundary $y^{[I]}(0)$ problem is also not the only one.

Theorem 1. If f(x)>0, continuous function α and β are positive constants, then the solution of the Cauchy problem (11)-(2) is in the form (4).

Theorem 2. Within the conditions of Theorem 1, (1), (5) there is a solution of the boundary problem in the form of (8). The solution to the border issue is not the only one.

CONCLUSION

The Cauchy and boundary problem is solved for the considered second formulation of the continuous multiplicative derivative equation. Arbitrary constants included in the general solution are determined with the help of initial or boundary conditions.

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