ROUGH STATISTICAL CONVERGENCE IN NEUTROSOPHIC NORMED SPACES

NÖTROSOFİK NORMLU UZAYLARDA ROUGH İSTATİSTİKSEL YAKINSAKLIK

Assist. Prof. Dr. Nazmiye Gönul Bilgin

Department of Mathematics, Faculty of Arts and Sciences, Zonguldak Bulent Ecevit University, Zonguldak, Türkiye

Geliş Tarihi / Received: 20.04.2022
Kabul Tarihi / Accepted: 20.05.2022
Araştırma Makalesi/Research Article
DOI: 10.38065/euroasiaorg.958

ABSTRACT

In this study, Neutrophic normed spaces, which is one of the popular mathematics topics of recent times, is discussed. The Neutrosophic approach, which argues that it is insufficient to evaluate the problems in daily life as just right and wrong, is based on the principle that the degree of indecision should be taken into account. Now, rough statistical convergence of triple sequences is defined in Neutrosophic normed spaces. Moreover, the important topological properties about to the set of cluster points of roughly statistical convergent triple sequences are given.

Keywords rough statistical convergence, neutrosophic normed spaces, triple sequences.

ÖZET

Bu çalışmada, son zamanların popüler matematik konularından biri olan Nötrosofik normlu uzaylar ele alınmıştır. Günlük yaşamdaki sorunları sadece doğru ve yanlış olarak değerlendirmenin yetersiz olduğunu savunan Nötrosofik yaklaşım, kararsızlığın derecesinin dikkate alınması gerektiği ilkesine dayanmaktadır. Bu yazıda, Nötrosofik normlu uzaylarda üçlü dizilerin rough istatistiksel yakınsaklığı tanımlanmıştır. Ayrıca, rough istatistiksel yakınsak üçlü dizilerin yığılma noktaları kümesiyle ilgili önemli topolojik özellikler verilmiştir.

Anahtar Kelimeler rough istatistiksel yakınsaklık, nötrosofik normlu uzaylar, üçlü diziler.

1. INTRODUCTION

The concept of fuzzy set was introduced in (Zadeh, 1965). A lot of research has been done in this field. Some of the recent ones are as follows. (Chandra Das, 2018). (Bilgin and Bozma, 2020), (Adhya and Deb Ray, 2022). Later, the intuitionistic fuzzy set concept was established in (Atanassov, 1986), which is an alternative approach to define a fuzzy set when the available information is not enough to define a fuzzy set. A few examples of studies on this concept are as follows. (Coskun, 2000), (Bilgin and Bozma, 2021), (Granados and Das, 2022). While intuitionistic fuzzy sets seem to adequately describe the uncertainty and lack of precision of the data, this is not the case, thanks to the approach introduced by (Smarandache, 1998) in recent years. Neutrosophic sets are constructed with the help of the membership function of indecision states.

Statistical convergence in Neutrosophic normed space is given by (Kirisci and Simsek, 2020). (Granados and Dhital, 2021) defined statistical convergence for double sequences on Neutrosophic normed space. Neutrosophic triplet normed space is given by (Sahin and Kargın, 2017). Later, many convergence types is carried to Neutrosophic normed space. One of them is rough convergence.

The rough convergence has been initially defined by (Phu, 2001) for normed spaces. (Aytar, 2008) used the concept of natural density to transfer the rough convergence to rough statistical convergence. (Malik and Maity, 2013) and (Dundar and Cakan, 2014), is defined rough convergence in double sequences.

The rough statistical convergence is defined using triple sequences in (Debnath and Subramanian, 2017). (Antal et all, 2021) studied rough statistical convergence in IFNS and gave main properties this convergence. The idea of rough convergence is quite interesting. Because it is well known that if a sequences is ordinarily convergent, its limit is unique. This property does not apply to rough convergence.

Neutrosophic normed space is preferred to study in this paper as it is a comprehensive mathematical subject to explain truth, falsity and uncertainty in daily life problems. After the studies in the literature mentioned above, the idea of transferring the concept of rough convergence of triple sequences to the theory of statistical convergence emerged, in this study, the subject of rough statistical convergence of triple sequences in neutrosophic spaces will be studied in order to fill the relevant deficiency in the literature.

2. PRELIMINARIES

A sequence (y_n) is called to be statistically convergent to y if for each $\varepsilon > 0$,

$$\delta(\{n \in \mathbb{N}: |y_k - y| \geq \varepsilon\}) = 0.$$

Then, the three-dimensional analogue of natural density is defined (see for example Sahiner et all, 2007) as:

A subset M of $\mathbb{N} \times \mathbb{N} \times \mathbb{N}$ is called to have natural density $\delta_3(M)$ where

$$\delta_3(M) = \lim_{u,v,w\to\infty} \frac{|M(u,v,w)|}{uvw}$$

exists. Here, $|M(u,v,w)|$ demonstrate the numbers of (m,n,o) in D where, $u \geq m, v \geq n \text{ and } w \geq o.$

(Sahiner et all, 2007) defined statistical convergence for triple sequence. (y_{mno}) is called to be statistical convergent to y if for all $\varepsilon > 0$,

$$\delta_3(\{(m,n,o) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N}: |y_{mno} - y| \geq \varepsilon\}) = 0.$$

In this case, it is denote with $st - \lim_{m,n,o\to\infty} y_{mno} = y.$

Now, the definition of rough convergence in the sense of Pringsheim defined using triple sequences by (Debnath and Subramanian, 2017). will be given as:

(y_{mno}) is named to be rough convergent to y demonstrated by $y_{mno} \xrightarrow{r} y$ such that for every $\varepsilon > 0$,

a $n_0 \in \mathbb{N}: m,n,o \geq n_0$ then $|y_{mno} - y| < r + \varepsilon$. Here, r is the roughness of degree.

The rough limit set of (y_{mno}) is demonstrated with $LIM^{r}_{mno} \coloneqq \{y: y_{mno} \xrightarrow{r} y\}$. It can be easily seen that rough limit set is not unique.

It's time to remind important concepts related to neutrophic normed space.

Definition 1.2 (Smarandache, 1998) Let $\mathcal{X} \neq \emptyset$, $\tau_{(\mathfrak{n},c)}(y), \varsigma_{(\mathfrak{n},u)}(y)$ and $\xi_{(\mathfrak{n},f)}(y)$ are the degrees of correctness, uncertainty and falsity. A neutrosophic set $\mathfrak{n}$ is in the next form: $\mathfrak{n} = \left\{\left(y, \tau_{(\mathfrak{n},c)}(y), \varsigma_{(\mathfrak{n},u)}(y), \xi_{(\mathfrak{n},f)}(y)\right): y \in \mathcal{X}\right\}$ where for all y in $\mathcal{X}$; $\tau_{(\mathfrak{n},c)}(y), \varsigma_{(\mathfrak{n},u)}(y)$ and $\xi_{(\mathfrak{n},f)}(y) \in [0,1]$, $0 \leq \tau_{(\mathfrak{n},c)}(y) + \varsigma_{(\mathfrak{n},u)}(y) + \xi_{(\mathfrak{n},f)}(y) \leq 2.$

Here, it should be noted that, $\varsigma_{(\mathfrak{n},u)}(s)$ is an independent component, $\tau_{(\mathfrak{n},c)}(y)$ and $\xi_{(\mathfrak{n},f)}(y)$ are dependent components.

Definition 1.3. (Kirisci and Simsek, 2020) Let $\mathcal{X}$ be a linear spaces, $\diamond$ and $\triangle$ demonstrate the continuous $\mathfrak{t}$ −norm and continuous $\mathfrak{t}$ −conorm on $\mathbb{R}$. The notation of Neutrosophic Normed is $\{((\mathfrak{o},\mathfrak{p}),\tau_{(\mathfrak{n},\mathfrak{c})}(\mathfrak{o},\mathfrak{p}),\varsigma_{(\mathfrak{n},\mathfrak{u})}(\mathfrak{o},\mathfrak{p}),\xi_{(\mathfrak{n},\mathfrak{f})}(\mathfrak{o},\mathfrak{p})):(\mathfrak{o},\mathfrak{p})\in\mathcal{X}\times(0,\infty)\}$ where $\tau_{(\mathfrak{n},\mathfrak{c})}$, $\varsigma_{(\mathfrak{n},\mathfrak{u})}$ and $\xi_{(\mathfrak{n},\mathfrak{f})}$ demonstrate the degree of correctness, uncertainty and falsity of $(\mathfrak{o},\mathfrak{p})$ on $\mathcal{X}\times(0,\infty)$ satisfies following conditions: For all $\mathfrak{o}_1,\mathfrak{o}_2\in\mathcal{X}$,

i. For every $\mathfrak{p}\in\mathbb{R}^+$ $\tau_{(\mathfrak{n},\mathfrak{c})}(\mathfrak{o},\mathfrak{p})+\varsigma_{(\mathfrak{n},\mathfrak{u})}(\mathfrak{o},\mathfrak{p})+\xi_{(\mathfrak{n},\mathfrak{f})}(\mathfrak{o},\mathfrak{p})\leq 2$,
ii. For every $\mathfrak{p}_1,\mathfrak{p}_2\in\mathbb{R}^+$,
$\tau_{(\mathfrak{n},\mathfrak{c})}(\mathfrak{o}_1,\mathfrak{p}_1)\diamond\tau_{(\mathfrak{n},\mathfrak{c})}(\mathfrak{o}_2,\mathfrak{p}_2)\leq\tau_{(\mathfrak{n},\mathfrak{c})}(\mathfrak{o}_1+\mathfrak{o}_2,\mathfrak{p}_1+\mathfrak{p}_2)$,
$\varsigma_{(\mathfrak{n},\mathfrak{u})}(\mathfrak{o}_1,\mathfrak{p}_1)\triangle\varsigma_{(\mathfrak{n},\mathfrak{u})}(\mathfrak{o}_2,\mathfrak{p}_2)\geq\varsigma_{(\mathfrak{n},\mathfrak{u})}(\mathfrak{o}_1+\mathfrak{o}_2,\mathfrak{p}_1+\mathfrak{p}_2)$,
$\xi_{(\mathfrak{n},\mathfrak{f})}(\mathfrak{o}_1,\mathfrak{p}_1)\triangle\xi_{(\mathfrak{n},\mathfrak{f})}(\mathfrak{o}_2,\mathfrak{p}_2)\geq\xi_{(\mathfrak{n},\mathfrak{f})}(\mathfrak{o}_1+\mathfrak{o}_2,\mathfrak{p}_1+\mathfrak{p}_2)$.
iii. For every $\mathfrak{p}\in\mathbb{R}^+$, $\tau_{(\mathfrak{n},\mathfrak{c})}(\mathfrak{o},\mathfrak{p})=1\Leftrightarrow\mathfrak{o}=0$, $\varsigma_{(\mathfrak{n},\mathfrak{u})}(\mathfrak{o},\mathfrak{p})=0\Leftrightarrow\mathfrak{o}=0$, $\xi_{(\mathfrak{n},\mathfrak{f})}(\mathfrak{o},\mathfrak{p})=0\Leftrightarrow\mathfrak{o}=0$,
iv. For each $p\neq 0$, $\tau_{(\mathfrak{n},\mathfrak{c})}(p\mathfrak{o},\mathfrak{p})=\tau_{(\mathfrak{n},\mathfrak{c})}\left(\mathfrak{o},\frac{\mathfrak{p}}{|p|}\right)$, $\varsigma_{(\mathfrak{n},\mathfrak{u})}(p\mathfrak{o},\mathfrak{p})=\varsigma_{(\mathfrak{n},\mathfrak{u})}\left(\mathfrak{o},\frac{\mathfrak{p}}{|p|}\right)$ and $\xi_{(\mathfrak{n},\mathfrak{f})}(p\mathfrak{o},\mathfrak{p})=\xi_{(\mathfrak{n},\mathfrak{f})}\left(\mathfrak{o},\frac{\mathfrak{p}}{|p|}\right)$.
v. $\tau_{(\mathfrak{n},\mathfrak{c})}(\mathfrak{o},\cdot)$ is continuous non-decreasing function, $\varsigma_{(\mathfrak{n},\mathfrak{u})}(\mathfrak{o},\cdot)$ and $\xi_{(\mathfrak{n},\mathfrak{f})}(\mathfrak{o},\cdot)$ are continuous non-increasing function,
vi. $\lim_{s\to\infty}\tau_{(\mathfrak{n},\mathfrak{c})}(\mathfrak{o},\mathfrak{p})=1$, $\lim_{s\to\infty}\varsigma_{(\mathfrak{n},\mathfrak{u})}(\mathfrak{o},\mathfrak{p})=0$ and $\lim_{s\to\infty}\xi_{(\mathfrak{n},\mathfrak{f})}(\mathfrak{o},\mathfrak{p})=0$.
vii. If $s\leq 0$, then $\tau_{(\mathfrak{n},\mathfrak{c})}(\mathfrak{o},\mathfrak{p})=0$, $\varsigma_{(\mathfrak{n},\mathfrak{u})}(\mathfrak{o},\mathfrak{p})=1$ and $\xi_{(\mathfrak{n},\mathfrak{f})}(\mathfrak{o},\mathfrak{p})=1$.

In this case, $(\mathcal{X},\tau_{(\mathfrak{n},\mathfrak{c})},\varsigma_{(\mathfrak{n},\mathfrak{u})},\xi_{(\mathfrak{n},\mathfrak{f})},\diamond,\triangle)$ is called Neutrosophic Normed Spaces. Here $\tau_{(\mathfrak{n},\mathfrak{c})}$ and $\varsigma_{(\mathfrak{n},\mathfrak{u})}$ are interdependent and $\xi_{(\mathfrak{n},\mathfrak{f})}$ is an independent components.

A few studies on the types of convergence in Neutrosophic spaces are as follows.(Khan et all, 2021),(Kisi, 2021) and (Gonul Bilgin, 2022). Now with the motivation of the studies done in e.g. (Antal et all, 2021), (Debnath and Subramanian, 2017) and (Kisi and Gurdal, 2022) it is possible to move on to the section where new definitions and theorems will be given.

3. MATHERIALS and METHODS

Definition 3.1 Let $(\mathcal{X},\tau_{(\mathfrak{n},\mathfrak{c})},\varsigma_{(\mathfrak{n},\mathfrak{u})},\xi_{(\mathfrak{n},\mathfrak{f})},\diamond,\triangle)$ be a Neutrosophic Normed Spaces, (y_{mno}) be a triple sequences. (y_{mno}) is called to be rough convergent to y for some $r\in\mathbb{R}^+$ such that every $\varepsilon>0$, there exits a $n_0\in\mathbb{N}$ and $0<\gamma<1$:for all $m,n,o\geq n_0$ if $\tau_{(\mathfrak{n},\mathfrak{c})}(y_{mno}-y,r+\varepsilon)>1-\gamma$, $\varsigma_{(\mathfrak{n},\mathfrak{u})}(y_{mno}-y,r+\varepsilon)<\gamma$ and $\xi_{(\mathfrak{n},\mathfrak{f})}(y_{mno}-y,r+\varepsilon)<\gamma$. Then, it is denote with $r-\lim_{m,n,k\to\infty}y_{mno}=y$.

Definition 3.2 Let (y_{mno}) be a triple sequences in $(\mathcal{X},\tau_{(\mathfrak{n},\mathfrak{c})},\varsigma_{(\mathfrak{n},\mathfrak{u})},\xi_{(\mathfrak{n},\mathfrak{f})},\diamond,\triangle)$. (y_{mno}) is named to be rough statistically convergent to y for some $r\in[0,\infty)$ such that every $\varepsilon>0$ and $0<\gamma<1$ if

$$\delta_3(\{(m,n,o)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\tau_{(\mathfrak{n},\mathfrak{c})}(y_{mno}-y,r+\varepsilon)\leq 1-\gamma \text{ or } \varsigma_{(\mathfrak{n},\mathfrak{u})}(y_{mno}-y,r+\varepsilon)\geq\gamma\ and$$

$$\xi_{(\mathfrak{n},\mathfrak{f})}(y_{mno}-y,r+\varepsilon)\geq\gamma\})=0.$$ Then, it is denote with $st-r-\lim_{m,n,k\to\infty}y_{mno}=y$.

Here, For $r=0$, rough statistical convergence compatible with the statistical convergence in $(\mathcal{X},\tau_{(\mathfrak{n},\mathfrak{c})},\varsigma_{(\mathfrak{n},\mathfrak{u})},\xi_{(\mathfrak{n},\mathfrak{f})},\diamond,\triangle)$. The rough statistical limit of a triple sequences may not be unique. Let rough statistical limit set of (y_{mno}) is denoted with;

$$St-r-LIM(y_{mno})=\{y: st-r-\lim_{m,n,k\to\infty}y_{mno}=y\}.$$

Definition 3.3 Let $(\mathcal{X},\tau_{(\mathrm{n,c})},\varsigma_{(\mathrm{n,u})},\xi_{(\mathrm{n,f})},\ \diamond,\ \triangle)$ be a Neutrosophic Normed Spaces, (y_{mno}) be a triple sequences. (y_{mno}) is called to be rough statistically bounded for some $r\in\mathbb{R}^+$ such that every $\varepsilon>0$ and $0<\gamma<1$ if there exists a $K>0$ such that

$$\delta_3(\{(m,n,o)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\tau_{(\mathrm{n,c})}(y_{mno},K)\leq 1-\gamma \text{ or } \varsigma_{(\mathrm{n,u})}(y_{mno},K)\geq\gamma\ \ and\ \xi_{(\mathrm{n,f})}(y_{mno},K)\geq\gamma\})=0.$$

Now, using these definitions, the following important theorems can be initiated for triple sequences in Neutrosophic normed spaces.

Lemma 3.1 Let $(\mathcal{X},\tau_{(\mathrm{n,c})},\varsigma_{(\mathrm{n,u})},\xi_{(\mathrm{n,f})},\ \diamond,\ \triangle)$ be a Neutrosophic Normed Spaces, (y_{mno}) be a triple sequences. For some non negative real number r, if $St-r-LIM(y_{mno})\neq\emptyset$ then (y_{mno}) is bounded sequnces in $(\mathcal{X},\tau_{(\mathrm{n,c})},\varsigma_{(\mathrm{n,u})},\xi_{(\mathrm{n,f})},\ \diamond,\ \triangle)$.

Proof Let (y_{mno}) be a triple sequences in $(\mathcal{X},\tau_{(\mathrm{n,c})},\varsigma_{(\mathrm{n,u})},\xi_{(\mathrm{n,f})},\ \diamond,\ \triangle)$ and some >0, $St-r-LIM(y_{mno})\neq\emptyset$. Then there exists y and $y\in St-r-LIM(y_{mno})$. For all $\varepsilon>0$ and $0<\gamma<1$, it is written
$\delta_3(\{(m,n,o)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\tau_{(\mathrm{n,c})}(y_{mno}-y,r+\varepsilon)\leq 1-\gamma \text{ or } \varsigma_{(\mathrm{n,u})}(y_{mno}-y,r+\varepsilon)\geq\gamma\ \ and\ \xi_{(\mathrm{n,f})}(y_{mno}-y,r+\varepsilon)\geq\gamma\})=0.$
So, (y_{mno}) is statistically bounded in $(\mathcal{X},\tau_{(\mathrm{n,c})},\varsigma_{(\mathrm{n,u})},\xi_{(\mathrm{n,f})},\ \diamond,\ \triangle)$.

Lemma 3.2 Let $(\mathcal{X},\tau_{(\mathrm{n,c})},\varsigma_{(\mathrm{n,u})},\xi_{(\mathrm{n,f})},\ \diamond,\ \triangle)$ be a Neutrosophic Normed Spaces, (y_{mno}) be a triple sequences. If (y_{mno}) is bounded sequnces in $(\mathcal{X},\tau_{(\mathrm{n,c})},\varsigma_{(\mathrm{n,u})},\xi_{(\mathrm{n,f})},\ \diamond,\ \triangle)$ then, for some non negative real number r, if $St-r-LIM(y_{mno})\neq\emptyset$.

Proof (y_{mno}) be a triple sequences in $(\mathcal{X},\tau_{(\mathrm{n,c})},\varsigma_{(\mathrm{n,u})},\xi_{(\mathrm{n,f})},\ \diamond,\ \triangle)$ and (y_{mno}) is bounded sequnces. For all $\varepsilon>0$ and $0<\gamma<1$ and some non negative real number r there exists a $K>0$ such that

$$\delta_3(\{(m,n,o)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\tau_{(\mathrm{n,c})}(y_{mno},K)\leq 1-\gamma \text{ or } \varsigma_{(\mathrm{n,u})}(y_{mno},K)\geq\gamma\ \ and\ \xi_{(\mathrm{n,f})}(y_{mno},K)\geq\gamma\})=0.$$

Let a set of the form

$$\mathfrak{M}=\left\{(m,n,o)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\tau_{(\mathrm{n,c})}\left(y_{mno},K\right)\leq 1-\gamma \text{ or } \varsigma_{(\mathrm{n,u})}\left(y_{mno},K\right)\geq\gamma\ and\ \xi_{(\mathrm{n,f})}\left(y_{mno},K\right)\geq\gamma\right\}$$

is defined. For $(m,n,o)\in\mathfrak{M}^c$, it is written $\tau_{(\mathrm{n,c})}(y_{mno},K)>1-\gamma$ and $\varsigma_{(\mathrm{n,u})}(y_{mno},K)<\gamma$, $\xi_{(\mathrm{n,f})}(y_{mno},K)<\gamma$. Furthermore,

$\tau_{(\mathrm{n,c})}(y_{mno},r+K)>1-\ \gamma$ and $\varsigma_{(\mathrm{n,u})}(y_{mno},r+K)<\gamma,\xi_{(\mathrm{n,f})}(y_{mno},r+K)<\gamma.$

So $0\in St-r-LIM(y_{mno})$. Hence $St-r-LIM(y_{mno})\neq\emptyset$.

In the following section, some topological properties of the set of $St-r-LIM(y_{mno})$ will be examined.

Theorem 3.1 Let (y_{mno}) is a triple sequences in $(\mathcal{X},\tau_{(\mathrm{n,c})},\varsigma_{(\mathrm{n,u})},\xi_{(\mathrm{n,f})},\ \diamond,\ \triangle)$ then $St-r-LIM(y_{mno})$ is closed sets.

Proof It is easy to show that $St-r-LIM(y_{mno})=\emptyset$, so let $St-r-LIM(y_{mno})\neq\emptyset$. Then, choosing a triple sequences (y_{mno}) and $\gamma'>0$, where $(1-\gamma)\diamond(1-\gamma)>1-\gamma'$, $\gamma\triangle\gamma<\gamma'$and $st-r-\lim_{m,n,k\to\infty}y_{mno}=y$. It will be shown that $y\in St-r-LIM(y_{mno})$. Let $\varepsilon>0$ and using rough statistical convergence; there exists a $n_0\in\mathbb{N}$ such that for $m,n,k\geq n_0$,

$\tau_{(\text{n,c})}\left(y_{mno}-y,\frac{\varepsilon}{3}\right)>1-\gamma$, $\varsigma_{(\text{n,u})}\left(y_{mno}-y,\frac{\varepsilon}{3}\right)<\gamma$ and $\xi_{(\text{n,f})}\left(y_{mno}-y,\frac{\varepsilon}{3}\right)<\gamma$.

If choosing $y_{m'n'o'}\in St-r-LIM(y_{mno})$ where $m',n',o'>n_0$ such that

$$\delta_3\left(\left\{(m,n,o)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\tau_{(\text{n,c})}\left(y_{mno}-y_{m'n'o'},r+\frac{\varepsilon}{3}\right)\le 1-\gamma \text{ or } \varsigma_{(\text{n,u})}\left(y_{mno}-y_{m'n'o'},r+\frac{\varepsilon}{3}\right)\ge\gamma \text{ and } \xi_{(\text{n,f})}\left(y_{mno}-y_{m'n'o'},r+\frac{\varepsilon}{3}\right)\ge\gamma\right\}\right)=0.$$

For $(k,l,s)\in\left\{(m,n,o)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\tau_{(\text{n,c})}\left(y_{mno}-y_{m'n'o'},r+\frac{\varepsilon}{3}\right)>1-\gamma \text{ or } \varsigma_{(\text{n,u})}(y_{mno}-y_{m'n'o'},r+\frac{\varepsilon}{3})<\gamma \text{ and } \xi_{(\text{n,f})}\left(y_{mno}-y_{m'n'o'},r+\frac{\varepsilon}{3}\right)<\gamma\right\}$. Furthermore, $\tau_{(\text{n,c})}(y_{kls}-y,r+\varepsilon)>(1-\gamma)\diamond(1-\gamma)>1-\gamma'$ and $\varsigma_{(\text{n,u})}(y_{kls}-y,r+\varepsilon)\le\gamma\vartriangle\gamma<\gamma'$, $\xi_{(\text{n,f})}(y_{kls}-y,r+\varepsilon)\le\gamma\vartriangle\gamma<\gamma'$. So,

$(k,l,s)\in\{(m,n,o)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\tau_{(\text{n,c})}(y_{mno}-y,r+\varepsilon)>1-\gamma \text{ or } \varsigma_{(\text{n,u})}(y_{mno}-y,r+\varepsilon)<\gamma$ *and* $\xi_{(\text{n,f})}(y_{mno}-y,r+\varepsilon)<\gamma\}$. Then,

$$\left\{(m,n,o)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\tau_{(\text{n,c})}\left(y_{mno}-y_{m'n'o'},r+\frac{\varepsilon}{3}\right)>1-\gamma,\varsigma_{(\text{n,u})}\left(y_{mno}-y_{m'n'o'},r+\frac{\varepsilon}{3}\right)<\gamma \text{ and } \xi_{(\text{n,f})}\left(y_{mno}-y_{m'n'o'},r+\frac{\varepsilon}{3}\right)<\gamma\right\}\subseteq\left\{(m,n,o)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\tau_{(\text{n,c})}\left(y_{mno}-y,r+\frac{\varepsilon}{3}\right)>1-\gamma,\varsigma_{(\text{n,u})}\left(y_{mno}-y,r+\frac{\varepsilon}{3}\right)<\gamma \text{ and } \xi_{(\text{n,f})}\left(y_{mno}-y,r+\frac{\varepsilon}{3}\right)<\gamma\right\}.$$

Thus, $\delta_3\{(m,n,o)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\tau_{(\text{n,c})}(y_{mno}-y,r+\varepsilon)\le 1-\gamma \text{ or } \varsigma_{(\text{n,u})}(y_{mno}-y,r+\varepsilon)\ge\gamma$ and $\xi_{(\text{n,f})}(y_{mno}-y,r+\varepsilon)\ge\gamma\}\le\delta_3\left\{(m,n,o)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\tau_{(\text{n,c})}\left(y_{mno}-y_{m'n'o'},r+\frac{\varepsilon}{3}\right)\le 1-\gamma \text{ or } \varsigma_{(\text{n,u})}y_{mno}-y_{m'n'o'},r+\frac{\varepsilon}{3}\right)\ge\gamma \text{ and } \xi_{(\text{n,f})}\left(y_{mno}-y_{m'n'o'},r+\frac{\varepsilon}{3}\right)\ge\gamma\right\}$.

Hence,

$$\delta_3\{(m,n,o)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\tau_{(\text{n,c})}(y_{mno}-y,r+\varepsilon)\le 1-\gamma \text{ or } \varsigma_{(\text{n,u})}(y_{mno}-y,r+\varepsilon)\ge\gamma \text{ and } \xi_{(\text{n,f})}(y_{mno}-y,r+\varepsilon)\ge\gamma\}=0.$$

Consequently, it is shown that $y\in St-r-LIM(y_{mno})$.

Definition 3.4 Let $(\mathcal{X},\tau_{(\text{n,c})},\varsigma_{(\text{n,u})},\xi_{(\text{n,f})},\diamond,\vartriangle)$ be a Neutrosophic Normed Spaces, (y_{mno}) be a triple sequences. For some $r\in[0,\infty)$, all $\varepsilon>0$ and $0<\gamma<1$,

$\delta_3\{(m,n,o)\in\mathbb{N}\times\mathbb{N}\times\mathbb{N}:\tau_{(\text{n,c})}(y_{mno}-z,r+\varepsilon)>1-\gamma$ *and* $\varsigma_{(\text{n,u})}(y_{mno}-z,r+\varepsilon)<\gamma$ and $\xi_{(\text{n,f})}(y_{mno}-z,r+\varepsilon)<\gamma\}\ne 0$, then, y is named rough statistical cluster point of (y_{mno}). It is denote with $st-r-cls$ point of (y_{mno}). Let $\mathcal{C}^{r}_{y_{mno}}$is demonstrated the set of every $st-r-cls$ point of (y_{mno}) in $(\mathcal{X},\tau_{(\text{n,c})},\varsigma_{(\text{n,u})},\xi_{(\text{n,f})},\diamond,\vartriangle)$.

Theorem 3.2 Let $(\mathcal{X},\tau_{(\text{n,c})},\varsigma_{(\text{n,u})},\xi_{(\text{n,f})},min,max)$ be a Neutrosophic Normed Spaces, (y_{mno}) be a triple sequences. Then, for some $r\in[0,\infty)$, all $\varepsilon>0$ and $0<\gamma<1$ the set $\mathcal{C}^{r}_{y_{mno}}$ is closed.

Proof Let $\mathcal{C}^{r}_{y_{mno}}\ne\emptyset$ be taken as the proof for $\mathcal{C}^{r}_{y_{mno}}=\emptyset$ is clear. Now, choosing $(z_{mno})\subseteq\mathcal{C}^{r}_{y_{mno}}$ and $r-\lim_{m,n,k\to\infty}z_{mno}=z$. If $z\in\mathcal{C}^{r}_{y_{mno}}$ is proven, the proof is complete. Using definition of rough convergence of sequences, for each $\varepsilon>0$ and $0<\gamma<1$, there exists $n_0\in\mathbb{N}$ such that for

$m,n,o > n_0$, $\tau_{(\mathfrak{n},\mathfrak{c})}\left(y_{mno} - z, \frac{\varepsilon}{3}\right) > 1-\gamma$, $\varsigma_{(\mathfrak{n},\mathfrak{u})}\left(y_{mno} - z, \frac{\varepsilon}{3}\right) < \gamma$ and $\xi_{(\mathfrak{n},\mathfrak{f})}\left(y_{mno} - z, \frac{\varepsilon}{3}\right) < \gamma$. Choosing $\tilde{n} \in \mathbb{N}$ where $\tilde{n} \geq n_0$. So, $\tau_{(\mathfrak{n},\mathfrak{c})}\left(y_{\tilde{n}} - z, \frac{\varepsilon}{3}\right) > 1-\gamma$, $\varsigma_{(\mathfrak{n},\mathfrak{u})}\left(y_{\tilde{n}} - z, \frac{\varepsilon}{3}\right) < \gamma$ and $\xi_{(\mathfrak{n},\mathfrak{f})}\left(y_{\tilde{n}} - z, \frac{\varepsilon}{3}\right) < \gamma$. Using $(z_{mno}) \subseteq \mathcal{C}^{r}_{y_{mno}}$, it is written $y_{\tilde{n}} \in \mathcal{C}^{r}_{y_{mno}}$. So

$$\delta_3\left(\left\{(m,n,o) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \tau_{(\mathfrak{n},\mathfrak{c})}\left(y_{mno} - y_{\tilde{n}}, r + \frac{\varepsilon}{3}\right) > 1-\gamma,\ \varsigma_{(\mathfrak{n},\mathfrak{u})}\left(y_{mno} - y_{\tilde{n}}, r + \frac{\varepsilon}{3}\right) < \gamma \text{ and } \xi_{(\mathfrak{n},\mathfrak{f})}\left(y_{mno} - y_{\tilde{n}}, r + \frac{\varepsilon}{3}\right) < \gamma\right\}\right) \neq 0.$$

Taking

$$(\widehat{m}, \hat{n}, \hat{o}) \in \left\{(m,n,o) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \tau_{(\mathfrak{n},\mathfrak{c})}\left(y_{mno} - y_{\tilde{n}}, r + \frac{\varepsilon}{3}\right) > 1-\gamma, \varsigma_{(\mathfrak{n},\mathfrak{u})}\left(y_{mno} - y_{\tilde{n}}, r + \frac{\varepsilon}{3}\right) < \gamma \text{ and } \xi_{(\mathfrak{n},\mathfrak{f})}\left(y_{mno} - y_{\tilde{n}}, r + \frac{\varepsilon}{3}\right) < \gamma\right\}.$$

Thus, it is written that

$$\tau_{(\mathfrak{n},\mathfrak{c})}\left(y_{\widehat{m}\hat{n}\hat{o}} - y_{\tilde{n}}, r + \frac{\varepsilon}{3}\right) > 1-\gamma,\quad \varsigma_{(\mathfrak{n},\mathfrak{u})}\left(y_{\widehat{m}\hat{n}\hat{o}} - y_{\tilde{n}}, r + \frac{\varepsilon}{3}\right) < \gamma \quad\text{and}\quad \xi_{(\mathfrak{n},\mathfrak{f})}\left(y_{\widehat{m}\hat{n}\hat{o}} - y_{\tilde{n}}, r + \frac{\varepsilon}{3}\right) < \gamma.$$

Then, similarly

$$\tau_{(\mathfrak{n},\mathfrak{c})}\left(y_{\widehat{m}\hat{n}\hat{o}} - z, r + \varepsilon\right) > 1-\gamma,\ \varsigma_{(\mathfrak{n},\mathfrak{u})}\left(y_{\widehat{m}\hat{n}\hat{o}} - z, r + \varepsilon\right) < \gamma \text{ and } \xi_{(\mathfrak{n},\mathfrak{f})}\left(y_{\widehat{m}\hat{n}\hat{o}} - z, r + \varepsilon\right) < \gamma.$$

Hence,

$$(\widehat{m}, \hat{n}, \hat{o}) \in \left\{(m,n,o) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \tau_{(\mathfrak{n},\mathfrak{c})}\left(y_{mno} - z, r + \varepsilon\right) > 1-\gamma, \varsigma_{(\mathfrak{n},\mathfrak{u})}\left(y_{mno} - z, r + \varepsilon\right) < \gamma \text{ and } \xi_{(\mathfrak{n},\mathfrak{f})}\left(y_{mno} - z, r + \varepsilon\right) < \gamma\right\}.$$

Then,

$$\delta_3\left(\left\{(m,n,o) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \tau_{(\mathfrak{n},\mathfrak{c})}\left(y_{mno} - y_{\tilde{n}}, r + \frac{\varepsilon}{3}\right) > 1-\gamma,\ \varsigma_{(\mathfrak{n},\mathfrak{u})}\left(y_{mno} - y_{\tilde{n}}, r + \frac{\varepsilon}{3}\right) < \gamma \text{ and } \xi_{(\mathfrak{n},\mathfrak{f})}\left(y_{mno} - y_{\tilde{n}}, r + \frac{\varepsilon}{3}\right) < \gamma\right\}\right) \leq \delta_3\left(\left\{(m,n,o) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \tau_{(\mathfrak{n},\mathfrak{c})}\left(y_{mno} - z, r + \frac{\varepsilon}{3}\right) > 1-\gamma,\ \varsigma_{(\mathfrak{n},\mathfrak{u})}\left(y_{mno} - z, r + \frac{\varepsilon}{3}\right) < \gamma \text{ and } \xi_{(\mathfrak{n},\mathfrak{f})}\left(y_{mno} - z, r + \frac{\varepsilon}{3}\right) < \gamma\right\}\right).$$

Using definition of natural density

$$\delta_3\left(\left\{(m,n,o) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \tau_{(\mathfrak{n},\mathfrak{c})}\left(y_{mno} - z, r + \frac{\varepsilon}{3}\right) > 1-\gamma,\ \varsigma_{(\mathfrak{n},\mathfrak{u})}\left(y_{mno} - z, r + \frac{\varepsilon}{3}\right) < \gamma \text{ and } \xi_{(\mathfrak{n},\mathfrak{f})}\left(y_{mno} - z, r + \frac{\varepsilon}{3}\right) < \gamma\right\}\right) \neq 0.$$

So, $z \in \mathcal{C}^{r}_{y_{mno}}$. This, completes the proof.

Theorem 3.3 Let (y_{mno}) is a triple sequences in $\left(\mathcal{X}, \tau_{(\mathfrak{n},\mathfrak{c})}, \varsigma_{(\mathfrak{n},\mathfrak{u})}, \xi_{(\mathfrak{n},\mathfrak{f})}, \min, \max\right)$ then the set $St - r - LIM\left(y_{mno}\right)$ is convex.

Proof

For $y_1, y_2 \in St - r - LIM(y_{mno}), \varepsilon > 0$ and some $0 < \alpha < 1$, it will be shown that $\left((1-\alpha)y_1 + \alpha y_2\right) \in St - r - LIM(y_{mno})$. Let $R, \widetilde{R}$ be defined as:

$$R = \left\{(m,n,o) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N} : \tau_{(\mathfrak{n},\mathfrak{c})}\left(y_{mno} - y_1, \frac{r+\varepsilon}{3(1-\alpha)}\right) \leq 1-\gamma \ \ or\ \ \varsigma_{(\mathfrak{n},\mathfrak{u})}\left(y_{mno} - y_1, \frac{r+\varepsilon}{3(1-\alpha)}\right) \geq \gamma \text{ and } \xi_{(\mathfrak{n},\mathfrak{f})}\left(y_{mno} - y_1, \frac{r+\varepsilon}{3(1-\alpha)}\right) \geq \gamma\right\},$$

$\tilde{R} = \left\{ (m,n,o) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \tau_{(\mathfrak{n},\mathfrak{c})}\left(y_{mno} - y_2, \frac{r+\varepsilon}{3\alpha}\right) \leq 1-\gamma \ \ or \ \ \varsigma_{(\mathfrak{n},\mathfrak{u})}\left(y_{mno} - y_2, \frac{r+\varepsilon}{3\alpha}\right) \geq \gamma \text{ and } \xi_{(\mathfrak{n},\mathfrak{f})}\left(y_{mno} - y_2, \frac{r+\varepsilon}{3\alpha}\right) \geq \gamma \right\}.$

Using $y_1, y_2 \in St - r - LIM(y_{mno})$, it is written $\delta_3(R) = \delta_3(\tilde{R}) = 0$. Let $(\hat{m}, \hat{n}, \hat{o}) \in R^c \cap \tilde{R}^c$, then

$$\tau_{(\mathfrak{n},\mathfrak{c})}(y_{mno} - ((1-\alpha)y_1 + \alpha y_2), r+\varepsilon) = \tau_{(\mathfrak{n},\mathfrak{c})}((1-\alpha)(y_{mno} - y_1) + \alpha(y_{mno} - y_2), r+\varepsilon)$$

$$\geq \min\left\{\tau_{(\mathfrak{n},\mathfrak{c})}\left((1-\alpha)(y_{mno} - y_1), \frac{r+\varepsilon}{2}\right), \tau_{(\mathfrak{n},\mathfrak{c})}\left(\alpha(y_{mno} - y_2), \frac{r+\varepsilon}{2}\right)\right\}$$

$$= \min\left\{\tau_{(\mathfrak{n},\mathfrak{c})}\left((y_{mno} - y_1), \frac{r+\varepsilon}{2(1-\alpha)}\right), \tau_{(\mathfrak{n},\mathfrak{c})}\left((y_{mno} - y_2), \frac{r+\varepsilon}{2\alpha}\right)\right\} > 1-\gamma,$$

$$\varsigma_{(\mathfrak{n},\mathfrak{u})}\left(y_{mno} - \left((1-\alpha)y_1 + \alpha y_2\right), r+\varepsilon\right) = \varsigma_{(\mathfrak{n},\mathfrak{u})}\left((1-\alpha)\left(y_{mno} - y_1\right) + \alpha\left(y_{mno} - y_2\right), r+\varepsilon\right)$$

$$\leq \max\left\{\varsigma_{(\mathfrak{n},\mathfrak{u})}\left((1-\alpha)(y_{mno} - y_1), \frac{r+\varepsilon}{2}\right), \varsigma_{(\mathfrak{n},\mathfrak{u})}\left(\alpha(y_{mno} - y_2), \frac{r+\varepsilon}{2}\right)\right\} < \gamma$$

and

$$\xi_{(\mathfrak{n},\mathfrak{f})}\left(y_{mno} - \left((1-\alpha)y_1 + \alpha y_2\right), r+\varepsilon\right) = \xi_{(\mathfrak{n},\mathfrak{f})}\left((1-\alpha)\left(y_{mno} - y_1\right) + \alpha\left(y_{mno} - y_2\right), r+\varepsilon\right)$$

$$\leq \max\left\{\xi_{(\mathfrak{n},\mathfrak{f})}\left((y_{mno} - y_1), \frac{r+\varepsilon}{2(1-\alpha)}\right), \xi_{(\mathfrak{n},\mathfrak{f})}\left(\alpha(y_{mno} - y_2), \frac{r+\varepsilon}{2}\right)\right\} < \gamma.$$

Then, it is written that

$\delta_3 \left(\{ (m,n,o) \in \mathbb{N} \times \mathbb{N} \times \mathbb{N} : \tau_{(\mathfrak{n},\mathfrak{c})}\left(y_{mno} - \left((1-\alpha)y_1 + \alpha y_2\right), r+\varepsilon\right) \leq 1-\gamma \text{ or } \varsigma_{(\mathfrak{n},\mathfrak{u})}\left(y_{mno} - ((1-\alpha)y_1 + \alpha y_2), r+\varepsilon\right) \geq 1-\gamma \text{ and } \xi_{(\mathfrak{n},\mathfrak{f})}\left(y_{mno} - ((1-\alpha)y_1 + \alpha y_2), r+\varepsilon\right) \geq 1-\gamma\}\right) = 0.$

Consequently, $((1-\alpha)y_1 + \alpha y_2) \in St - r - LIM(y_{mno})$. So, it is shown that the set $St - r - LIM(y_{mno})$ is convex.

Theorem 3.4 Let $(\mathcal{X}, \tau_{(\mathfrak{n},\mathfrak{c})}, \varsigma_{(\mathfrak{n},\mathfrak{u})}, \xi_{(\mathfrak{n},\mathfrak{f})}, \min, \max)$ be a Neutrosophic Normed Spaces, (y_{mno}) be a triple sequences. For some $r > 0$, $0 < \gamma < 1$ and a $w \in \mathcal{X}$(fixed), let

$\sigma(w,\gamma,r) = \{y_{mno} : \tau_{(\mathfrak{n},\mathfrak{c})}(y_{mno} - w, r) > 1-\gamma, \varsigma_{(\mathfrak{n},\mathfrak{u})}(y_{mno} - w, r) \leq \gamma, \xi_{(\mathfrak{n},\mathfrak{f})}(y_{mno} - w, r) \leq \gamma\}$

and

$\overline{\sigma(w,\gamma,r)} = \{y_{mno} : \tau_{(\mathfrak{n},\mathfrak{c})}(y_{mno} - w, r) \geq 1-\gamma, \varsigma_{(\mathfrak{n},\mathfrak{u})}(y_{mno} - w, r) < \gamma, \xi_{(\mathfrak{n},\mathfrak{f})}(y_{mno} - w, r) < \gamma\}.$

Then,

$$\mathcal{C}^{r}_{y_{mno}} = \bigcup_{w \in \mathcal{C}_{y_{mno}}} \overline{\sigma(w,\gamma,r)}$$

where $\mathcal{C}_{y_{mno}}$ is denote the set of ordinary statistical cluster point of (y_{mno}).

Proof

Let $y \in \bigcup_{w\in\mathcal{C}_{y_{mno}}} \overline{\sigma(w,\gamma,r)}$. For some $r > 0$ and given $\gamma \in (0,1)$ there exists $w \in \mathcal{C}_{y_{mno}}$ such that $\tau_{(\text{n,c})}(w - y, r) > 1-\gamma$, $\varsigma_{(\text{n,u})}(w-y,r) < \gamma$ and $\xi_{(\text{n,f})}(w-y,r) < \gamma$.
Also, for $\bar{r} > 0$ using $w \in \mathcal{C}_{y_{mno}}$, then there exists a set
$P = (m,n,o) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}\tau_{(\text{n,c})}(y_{mno} - w, \bar{r}) > 1-\gamma, \varsigma_{(\text{n,u})}(y_{mno} - w,\bar{r}) < \gamma, \xi_{(\text{n,f})}(y_{mno} - w, \bar{r}) < \gamma$
and $\delta_3(P) \neq 0$. Hence for $(m,n,o) \in P$,
$\tau_{(\text{n,c})}(y_{mno} - w, r + \bar{r}) \geq \min\{\tau_{(\text{n,c})}(y_{mno} - w, \bar{r}), \tau_{(\text{n,c})}(w-y,r)\} > 1-\gamma$,
$\varsigma_{(\text{n,u})}(y_{mno} - w, r + \bar{r}) \leq \max\{\varsigma_{(\text{n,u})}(y_{mno} - w, \bar{r}), \varsigma_{(\text{n,u})}(w-y,r)\} < \gamma$
and
$\xi_{(\text{n,f})}(y_{mno} - w, r + \bar{r}) \leq \max\{\xi_{(\text{n,f})}(y_{mno} - w, \bar{r}), \xi_{(\text{n,f})}(w-y,r)\} < \gamma$.
From hence,
$\delta_3\Big\{\Big((m,n,o) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}: \tau_{(\text{n,c})}(y_{mno} - y, r + \bar{r}) > 1-\gamma, \varsigma_{(\text{n,u})}(y_{mno} - y, r+\bar{r}) < \gamma$ and $\xi_{(\text{n,f})}(y_{mno} - y, r + \bar{r}) < \gamma\Big)\Big\} \neq 0$.
Thus, $y \in \mathcal{C}^{r}_{y_{mno}}$ and then $\bigcup_{w\in\mathcal{C}_{y_{mno}}} \overline{\sigma(w,\gamma,r)} \subseteq \mathcal{C}^{r}_{y_{mno}}$.
It can be easily shown from the definition of cluster that $\mathcal{C}^{r}_{y_{mno}} \subseteq \bigcup_{w\in\mathcal{C}_{y_{mno}}} \overline{\sigma(w,\gamma,r)}$

Theorem 3.5

Let (y_{mno}) be a triple sequences in $(\mathcal{X}, \tau_{(\text{n,c})}, \varsigma_{(\text{n,u})}, \xi_{(\text{n,f})}, min, max)$ and (y_{mno}) is statistically convergent to y.For some $r > 0$, there exists $\gamma \in (0,1)$ such that $St - r - LIM(y_{mno}) = \overline{\sigma(y,\gamma,r)}$

Proof

For $\ddot{r} > 0$, using statistical convergence of (y_{mno}), there exists

$T = \{(m,n,o) \in \mathbb{N}\times\mathbb{N}\times\mathbb{N}: \tau_{(\text{n,c})}(y_{mno} - y, \ddot{r}) \leq 1-\gamma \; or \; \varsigma_{(\text{n,u})}(y_{mno} - y, \ddot{r}) \geq \gamma \; and \; \xi_{(\text{n,f})}(y_{mno} - y, \ddot{r}) \geq \gamma\}$ and $\delta_3(T) = 0$. Let $s \in \overline{\sigma(y,\gamma,r)}$ and for $(m,n,o) \in T^c$,

$\tau_{(\text{n,c})}(y_{mno} - s, r + \ddot{r}) > 1-\gamma$, $\varsigma_{(\text{n,u})}(y_{mno} - s, r+\ddot{r}) < \gamma$ and $\xi_{(\text{n,f})}(y_{mno} - s, r + \ddot{r}) < \gamma$.

That is $s \in St - r - LIM(y_{mno})$. So, $\overline{\sigma(y,\gamma,r)} \subseteq St - r - LIM(y_{mno})$. Furthermore,
$St - r - LIM(y_{mno}) \subseteq \overline{\sigma(y,\gamma,r)}$. Consequently, $St - r - LIM(y_{mno}) = \overline{\sigma(y,\gamma,r)}$.

4. CONCLUSION

We have carried the concept of rough statistical convergence, defined in intuitionistic fuzzy normed spaces, to Neutrosophistic normed spaces using triple sequences. So, we have extended some well-known important results. The important topological properties of the set of rough statistical limit points are given.

Acknowledgement

I would like to thank the editors, referees for their careful reading and the authors of the works included in the references and used while preparing the study.

Funding

No funding was received.

REFERENCES

Adhya, S. & Deb Ray, A. (2022). On weak G-completeness for fuzzy metric spaces. Soft Computing, 1-7.

Antal, R. Chawla, M. & Kumar, V. (2021). Rough statistical convergence in intuitionistic fuzzy normed spaces. Filomat, 35(13), 4405-4416.

Atanassov, K. (1986) Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1), pp.87–96.

Aytar, S. (2008), Rough statistical convergence, Numer. Funct. Anal. Optimiz. 29(3-4) 291–303.

Bilgin, N. G. & Bozma, G. (2020). On Fuzzy n-Normed Spaces Lacunary Statistical Convergence of order α . i-Manager's Journal on Mathematics, 9(2), 1.

Bilgin, N. G. & Bozma, G. (2021). Fibonacci Lacunary Statistical Convergence of order γ in IFNLS.

Bilgin, N. G., (2022). New Kinds of Statistical Convergence on Neutrosophic Normed Spaces, Math Sciences in International Research, Egitim Publishing.

Chandra Das, P. (2018). Fuzzy normed linear sequence space. Proyecciones (Antofagasta), 37(2), 389-403.

Coskun, E., (2000), Systems on intuitionistic fuzzy special sets and intuitionistic fuzzy special measures, Information Sciences, 128, 105-118.

Debnath, S. & Subramanian, N. (2017). Rough statistical convergence on triple sequences. Proyecciones (Antofagasta), 36(4), 685-699.

Granados, C. & Dhital, A. (2021). Statistical convergence of double sequences in neutrosophic normed spaces. Neutrosophic Sets and Systems, 42, 333-344.

Granados, C. & Das, S. (2022). On (λ, μ, ζ)-Zweier ideal convergence in intuitionistic fuzzy normed spaces. Yugoslav Journal of Operations Research.

Kirisci M. and Simsek N., (2020) Neutrosophic normed spaces and statistical convergence,The Journal of Analysis, 1-15.

Kisi. O. & Gurdal, V. (2022). On Triple Difference Sequences of Real Numbers in Neutrosophic Normed Spaces. Communications in Advanced Mathematical Sciences, 5(1), 35-45.

Kisi, O. (2021), Ideal convergence of sequences in neutrosophic normed spaces. Journal of Intelligent & Fuzzy Systems, (Preprint), 1-10.

Malik P., M. Maity, (2013), On rough convergence of double sequence in normed linear spaces, Bull. Allah. Math. Soc. 28(1), 89–99.

Phu H.X., (2001), Rough convergence in normed linear spaces, Numer. Funct. Anal. Optim. 22(1-2) 199–222.

Smarandache, F. Neutrosophy, Neutrosophic Probability, Set, and Logic, ProQuest Information and Learning. Ann Arbor, Michigan, USA, 1998.

Sahin, M. & Kargın, A. (2017). Neutrosophic triplet normed space. Open Physics, 15(1), 697-704.

Sahiner, A. Gurdal, M. & Duden, F. K. (2007). Triple sequences and their statistical convergence. Selcuk J. Appl. Math. 8(2), 49-55.

Zadeh, L.A. (1965) Fuzzy sets, Information and Control, Vol. 8, No. 3, pp.338–356.